Universal Gravitational Potential Energy

Consider the case of dropping a ball from height $h_o$ near the surface of the earth.

Assume that the force of gravity $mg$ stays nearly constant. We say that the work done by gravity is the area under the $F_{\text{grav}}$ vs $h$ graph, which is $mgh_o - mgh_f$.

But in general: $W = \Delta K = -\Delta U_g$

and $W = \text{Area} = mgh_o - mgh_f$

so $-\Delta U_g = mgh_o - mgh_f$

or $U_{go} - U_{gf} = mgh_o - mgh_f$.

It makes sense to say

$U_{go} = mgh_o$

and $U_{gf} = mgh_f$

This leads to the familiar result that

$U_g = mgh$

where $h$ is measured from wherever you want: you choose the reference level.

What if we want to use the conservation of energy approach someplace other than on the surface of the earth? What if we leave earth’s surface and head out into space? Clearly, we can’t use $U_g = mgh$ because that formulation assumes the force of gravity is constant. Let’s follow the same approach we used above.
Consider the case of dropping a ball from an initial position \( r_0 \) pretty far out in space. Let the ball fall to a position \( r_f \). Let’s find the work done by gravity in going from \( r_0 \) to \( r_f \). We say that the work done by gravity is the area under the \( F_{\text{grav}} \) vs \( r \) curve, which is:

\[
W = \int_{r_0}^{r_f} F_{\text{grav}} \, dr = \int_{r_0}^{r_f} (- \frac{GMm}{r^2}) \, dr = \frac{GMm}{r_f} - \frac{GMm}{r_0}
\]

But \( W = \Delta K = -\Delta U_g \)

So \( -(U_{gf} - U_{go}) = GMm/r_f - GMm/r_0 \).

It makes sense to say \( U_{go} = -GMm/r_0 \), and \( U_{gf} = -GMm/r_f \).

This leads in general to the idea that

\[
U_g = -\frac{GMm}{r}
\]

where \( r \) is measured from the center of mass \( M \).

Note: Don’t worry too much about those pesky minus signs: they are a pain!

Note: If we start from infinite distance, \( r_o = \infty \), and \( U_{go} = 0 \). What we are doing here is putting our reference level \( (U_g = 0) \) at \( \infty \). Anything closer than \( \infty \) will have to have gravitational potential energy less than 0, so that potential energy will have to be negative!

Now let’s put this idea to use.
Prob 1: How fast will an apple be falling when it hits the earth if it is dropped from a distance of 1.0 earth radii above the surface as shown? Use the conservation of energy approach. $M_{\text{earth}} = 6.0 \times 10^{24}$ kg and $R_{\text{earth}} = 6.4 \times 10^6$ m.

Step 1: Sketch the apple in its final position. Label the initial and final positions, and make sure both locations have position arrows from the center of the earth. Label the velocities in each position.

Step 2: Assume $W_{nc} = 0$ (ignore atmospheric effects.) Apply the conservation of mechanical energy. Use our newfound gravitational potential energy.

Step 3: solve for $v_f$.

Note: your solution for $v_f$ will be the same as the speed of an object required for low earth orbit! If you could make a frictionless ramp with the right shape, you could “drop” an object into orbit. Sketch it!
Prob 2: What if you were to drop the apple in the problem above from an infinite distance away from the earth? Find the impact speed on earth in this case. Follow the same steps you did in prob 1.
Prob 3: Escape speed. Calculate the minimum speed you would have to throw the apple with to launch it from the surface of the earth so it will never return. In other words, how fast will you have to throw the apple so that when it gets infinitely far from earth, it will just barely come to rest. This is called “escape speed.”

Write down the general formula for escape speed from an object in terms of the objects mass and radius:

$$V_{esc} =$$